Temporal Difference Methods

for Control

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Difference between prediction and control in pseudocode

tabular TD(0) for q_π: $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right]$

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

What would you modify in the above to get the pseudo code for TD(0)?

Q-learning and on-policy vs. off-policy

SARSA: $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right]$

Q-learning: $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \max_b Q(S_{t+1}, b) - Q(S_t, A_t)\right]$

Notice what the target of the update represents and whether the underlying policy of that matches the behavior policy

On-policy constant-α MC: $V^{MC}(S_t) \leftarrow V^{MC}(S_t) + \alpha \left[\underbrace{G_t}_{\textbf{target}} - V^{MC}(S_t)\right]$

Off-policy constant-α MC: $V^{MC}(S_t) \leftarrow V^{MC}(S_t) + \alpha \left[\rho_{t:T-1} G_t - V^{MC}(S_t)\right]$

Difference between Q-learning and SARSA in pseudocode

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Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

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Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
    $S \leftarrow S'$
  until S is terminal

$t = 0$ | **choose action** | **Take action & observe** | **choose action** | **Take action & observe** | **choose action** | **Take action & observe** | **...**

Time $t \rightarrow$

Where would you put SARSA updates? **Where would you put Q-learning updates?**

What happens if we switch the time of update for both?

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Say we make the SARSA update after taking the next action. Is it still same or correct?

Say we make the Q-learning update before taking action. Is it still same or correct?

Modify the Q-learning Pseudocode minimally to get SARSA

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

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The equivalence of two SARSA algorithms breaks in the real world

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Question:

What will be an off-policy TD(0) update for q_π?