CMPUT 397
Worksheet 1
January 8, 2020

1. Compute the partial derivatives of $f(x, y) = x\ln(y)$ with respect to (x, y). The partial derivatives are simply the derivatives for each variable, assuming the others are fixed. Computing each partial derivative uses the exact same rules as derivatives for single variables.

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix}$$

2. Consider the following game: you roll two (fair) 6-sided dice.

(a) What is the expected value of the sum of the two dice?

(b) What is the variance of the sum of the two dice?

You win \$1 if the sum of the dice roll is 2, 5, 7, 8 or 11. Otherwise, you lose \$1.

(a) What is the expected value of the winnings for playing this game. In other words, how much money are you expected to gain (or lose).

(b) What is the variance of the winnings for this game?

(c) Would you play this game as stated above? How about if the amount won or lost was \$100? How about \$1000?

3. Suppose that in a lottery you have 0.01% chance of winning and the prize is \$1000. The ticket to enter the lottery costs you \$10. What is the expected amount you would earn, when buying a ticket for this lottery?

4. Adam and Martha propose a simple dice game to you. You can throw a die up to two times, and they will reward you with the amount equivalent to the face value of the die. If you throw a die once and 3 comes up, you can choose to take \$3 or throw again. If you choose to throw again and 2 comes up, you earn only \$2. The amount you earn is not additive and you only earn the amount of your last roll.

(a) Suppose in your first roll, the dice comes up as a 1. What is the expected amount you would earn in your second roll?

(b) For what values in your first roll should you re-roll the die?

(c) What is the expected amount you would earn in this game if you play optimally?

5. Prove the tower property
$$E[X] = E\left[E[X|Y]\right].$$

Hint:
$E[X|Y]$ is a random variable and can be thought of as a function $g(Y)$ of Y.

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6. We say an estimator is unbiased if its expected value equals the true value of the parameter being estimated. For example, if we roll a die 10 times and denote the value of the ith roll to be X_i, then the sample mean (average roll) is an unbiased estimator of the true mean since

$$E\left[\frac{\sum_{i=1}^{10} X_i}{10}\right] = E[X].$$

Where $E[X]$ is the expected value of a roll.

Suppose you would like to estimate what the expected roll of a fair die is but you only have a loaded die with probability mass function $\tilde{p}(x)$. Show that $Y = X\frac{1/6}{\tilde{p}(X)}$ is an unbiased estimator of the mean for a roll with a fair die, where X is the outcome of the loaded die.